

Notes: Related Rates

Pebble Problem:

If you drop a stone into a body of water ripples form across the surface of the water. Suppose you are told that the radius of a ripple is increasing at a rate of 6 inches per second. What is the rate of change in the area enclosed by the ripple 2 seconds after the pebble is dropped?

The Blimp Problem

A Blimp travelling overhead is tied to a rope. The rope is let out at a rate of 3 feet per second. Assuming the blimp remains at a constant altitude of 800 feet, how fast is the blimp moving when 1000 feet of rope have been let out?

Board Examples:

1. a) Assume that the radius r of a sphere is a differentiable function of t and let V be the volume of a sphere. Find an equation that relates $\frac{dV}{dt}$ and $\frac{dr}{dt}$.

1. b) Assume that the radius r and the height h of a cone are differentiable functions of t and let V be the volume of the cone. Find an equation that relates $\frac{dV}{dt}$, $\frac{dr}{dt}$, and $\frac{dh}{dt}$.

2. How fast does the radius of a spherical bubble change when you blow air into it at the rate of $10 \text{ cm}^3 / \text{s}$?

3. How fast does the water level drop when a cylindrical tank is drained at the rate of 3 liters/s?

4. A hot-air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at a rate of 0.14 radians per minute. How fast is the balloon rising?

5. Water runs into a conical tank at the rate of 9 ft^3 per minute. The tank stands vertex down and has a height of 10 feet and a base radius of 5 feet. How fast is the water level rising when the water is 6 feet deep?

6. A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 miles north of the intersection and the car is 0.8 miles to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

The Ladder Problem:

When a ladder slides down a wall the rate at which it falls downward is not necessarily equal to the rate at which the base of the ladder moves away from the wall. Suppose you are given a 13-foot ladder and you are told that the ladder is moving 6 feet per second away from the wall when the ladder is 12 feet from the wall. What is the rate of change in the y -direction?